

# A RELATIVISTIC AND AUTONOMOUS NAVIGATION SATELLITE SYSTEM

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A relativistic positioning system has been proposed by Bartolomé Coll in 2002. Since then, several groups have developed this topic with different approaches. I will present a work done in collaboration with Ljubljana University and the ESA Advanced Concepts Team. We developed a concept, Autonomous Basis of Coordinates, in order to take advantage of the full autonomy of a satellite constellation for navigation and positioning, by means of satellite inter-links. I will present the advantages of this new paradigm and a number of potential applications for reference systems, geophysics and relativistic gravitation.

## 1 Relativistic Positioning Systems (RPS)

The first proposal for a relativistic positioning system is SYPOR (“Système de Positionnement Relativiste”), proposed by Bartolomé Coll in 2002<sup>1</sup>. It is an alternative to the scheme of usual positioning systems. The idea is to give the constellation of satellites the possibility to constitute by itself a *primary* and *autonomous* positioning system, without any *a priori* realization of a terrestrial reference frame.

The relativistic positioning system is defined with the introduction of *emission coordinates*, which contain dynamical information of the satellite constellation. They have been reintroduced recently by several articles<sup>2,3,4,5</sup>. The definition of these coordinates is rather simple, but they are a very powerful tool in general relativity. Let us define four particles  $a = 1, 2, 3, 4$  coupled to general relativity. Along their worldlines  $C_a$ , one defines four one-parameter families of future null cones  $N_a(\tau^a)$  which are parametrized by proper time. The intersection of four future null cones  $N_a(\tau^a)$  from four worldlines  $C_a$  defines an event with emission coordinates  $(\tau^1, \tau^2, \tau^3, \tau^4)$ . Then, a user receiving four electromagnetic signals broadcasting the proper time of four satellites knows its position in this particular coordinate system.

RPS have been studied with different approaches these last years<sup>6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21</sup>. The purpose of this proceeding is to present an approach developed by a collaboration between the Advanced Concepts Team of the European Space Agency, the department of physics of Ljubljana University and SYRTE/Paris Observatory/UPMC.

## 2 From emission to global coordinates

A GNSS is a system of satellites emitting precise timing signals for the purpose of providing a local coordinate basis in space-time. In order to determine his space-time position with respect to this basis, an observer must receive four proper times emitted by four different satellites, and be able to calculate the local coordinates of the four satellites as a function of their emission coordinates.

We studied the use of emission coordinates theoretically and in practical scenarii by numerical simulations<sup>16,20,21</sup>. We provided mathematical tools to translate emission coordinates into space-time coordinates of the observer. A local Schwarzschild coordinate system was introduced as an idealized prototype of space-time in the vicinity of the Earth. The problem of connecting the local Schwarzschild frame to the global inertial frame is well understood in the framework of classical non-relativistic gravitational perturbation theory, but remains to be done in a general relativistic framework.

Analytic solutions for light-like and time-like geodesics were obtained in order to implement two algorithms: (i) an algorithm that calculates emission coordinates corresponding to the local Schwarzschild coordinates of a user, and (ii) the “reverse” algorithm that calculates space-time coordinates of a user from its emission coordinates. In a first approach, we assumed that orbital parameters of satellites are known. We have shown that the use of a fully relativistic code in GNSS offers a very promising alternative to the use of post-Newtonian approximations, and presents no technical obstacle.

The effects of non-gravitational perturbations have been studied. We have shown that the only yardstick of a GNSS is the clock, which provides absolute position both in space and time to any accuracy and stability allowed by noise and clock drifts. Clock drifts, adding up after some time, would result in considerable error in absolute position in space if the the clocks were not controlled. In current positioning scheme, correcting the clocks needs a constant monitoring of the satellites via Earth telemetry and the precise realization of a terrestrial reference frame. However, we realized that a GNSS constellation is also a very precise clock of its own, since orbital periods of its satellites are accurate constants of motion. Therefore, we proposed to use the dynamical information given by mutual timing between satellites to improve the long term phase stability of onboard clocks, as well as to improve the precision of constants of motion of the constellation. This proposal led us to define the concept of Autonomous Basis of Coordinates (ABC). Within such a scheme, we have shown that it is possible to correct the clocks to a level considerably surpassing the classical scheme, which is limited in accuracy by stochastic components of Earth dynamics.

## 3 Autonomous Basis of Coordinates (ABC)

In a GNSS constellation with more than four satellites, more than four emission coordinates are received by an observer: the positioning problem is over-determined. In order for the local basis to be self consistent, all combinations of emission coordinates, received at any event in space-time, must give the same four local coordinates for this event. The main constraint on self-consistency of a GNSS system comes from the precision of constants of motion. In order to adress this problem, the concept of Autonomous Basis of Coordinates (ABC) is introduced in Čadež *et al.*<sup>20</sup>. We propose that the constants of motion be determined and checked internally by the GNSS system in such a way that each satellite checks its own position as any other observer with respect to all the other satellites: in addition to emitting its proper time, each satellite also receives other satellite’s emission coordinates and makes its information available to the central GNSS control<sup>22,23</sup>.

The ABC concept aims to describe in a coherent frame both the dynamics of non-interacting

test bodies transmitting emission coordinates and the propagation of electromagnetic waves providing those coordinates. It uses the fact that both light and test bodies trajectories are geodesics that can be derived from the same Hamiltonian. It provides a means to translate dynamical information into the conventional representation based on local frames. Dynamical information, expressed in terms of emission coordinates, gives direct information about the Riemannian structure of space-time, and thus allows the construction of a local frame with coordinates and metric that provides a precise definition of equations of motion. We call the reference system and coordinates built via the ABC concept the *ABC reference system* and the *ABC coordinates*.

Let us use the nomenclature introduced by J. Kovalevsky and I. Mueller<sup>24</sup> to describe the ABC reference system:

**Concept:** the ABC coordinate system is built such that dynamics is consistent; dynamics is given by a Hamiltonian, that both describe space-time geometry and non-gravitational forces.

**Physical structure:** the reference system is physically materialized by a constellation of satellites in Earth orbit and inter-satellite links. Light and satellite geodesics create a physical space-time web that probe the space-time geometry.

**Modelling:** the model characterizes a particular choice of the Hamiltonian. We have studied three particular Hamiltonians<sup>20</sup>: Minkowski, Kepler and Schwarzschild. The ultimate goal is to obtain a Hamiltonian containing a complete description of all known gravitational and non-gravitational perturbations. This is the purpose of the Slovenian PECS/ESA project: “Relativistic global navigation system”<sup>a</sup>.

**Realization:** A realization of the reference system needs the implementation on future GNSS constellation of inter-satellite links, which is now under study<sup>22,23</sup>. We have done a simulation<sup>16,20,21</sup> for some specific idealized space-time geometries and have discovered some generic properties of ABC systems, as robustness of recovering constants of motion with respect to noise in the data, consistency of description with redundant number of satellites, the possibility to use the constellation as a clock with long term stability and the possibility to use perturbation theory to refine the Hamiltonian toward a better long term dynamical prediction. For example, we have shown an internal consistency of Galileo satellites positions at the millimetre level after only four orbits ( $\sim 36$  hours) with 200 data points (one point every 10 mn). The accuracy of constants of motion is expected to increase with time, when more data will become available to evaluate smaller and smaller discrepancies between dynamic prediction and dynamic observations provided by exchange of emission coordinates between satellites.

## 4 Applications in geophysics and relativistic gravimetry

Dynamics of bodies and light in a given space-time is unique to the geometry of this space-time. Therefore, geometry can in principle be determined on the basis of dynamical information and vice versa, dynamics can be predicted with an accuracy limited, in principle, only by the accuracy of geometric information. Thus the GNSS with inter-satellite links is a new type of gravimeter, we call it *Riemannian gravimeter*, that creates a space-time web with light and satellite geodesics that “scan” the space-time geometry around Earth.

The accuracy of an ABC reference system, realized with Galileo satellites, would increase with the accuracy of geometric information derived from dynamics. However, the relation of such an ABC reference system to a celestial reference system is not trivial, since the ABC reference system is gauged with the local geometry of the part of space-time where satellites move, while signals from distant quasars travel long distances across the universe and are affected by the intervening curvature of spacetime. Thus, the relation between the ABC reference frame and a

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<sup>a</sup><http://www.esa.int/SPECIALS/PECS/index.html>

celestial reference frame could, in principle, reveal important new information about the way in which the local geometry is integrated into the global arena of space-time. A discrepancy between the two frames could also reveal a violation of the equivalence principle, if non-gravitational perturbations as solar pressure can be modelled or measured accurately.

The possibility to define an extremely precise ABC reference frame is also very interesting for geophysics. A sub-millimetre level of accuracy of satellite positions would eventually allow comparable position accuracy on Earth surface, at least statistically, by properly averaging positions obtained by ground based GNSS receivers. Below millimetre level of accuracy, the shape of Earth and absolute positions of markers on the ground would certainly elucidate many important phenomena about our planet Earth. For example, a much deeper understanding of interior structure of the Earth could be reached by studying Earth and ocean tides. Continental drift would be measured with a precision, that could possibly be sufficient to model changing strain and stress in the Earth crust and eventually lead to earthquake prediction. Gravitational potential differences and driving ocean currents could also be detected, allowing us to study ocean dynamics at the same level of precision as today's meteorology understands dynamics of atmosphere.

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